IOSR Journal of Mechanical and Civil Engineering (IOSR-JMCE)
e-ISSN: 2278-1684,p-ISSN: 2320-334X,
PP 108-120
www.iosrjournals.org

Displacement Field In A Solid Layer Embedded In An Infinite Fluid

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Abstract*: The work done is a step towards analyzing the pressure field inside a sonar dome. Acoustic waves from afar are plane when they reach the dome but the wave that is transmitted into the dome is not plane. This occurs because the dome is doubly curved and the angle of incidence is a function of the coordinates of the dome. In this paper, a ray approach is presented to determine the acoustic field when a plane acoustic wave is incident on a solid layer embedded in an infinite fluid. Ray theory is used to determine the reflected and transmitted waves and the field inside the solid layer.*

Index Terms*: acoustics, sonar dome, ray theory, wave theory.*

I. Introduction

THE work done is a step towards analyzing the pressure field inside a sonar dome. Electro-acoustic transducers that convert electrical energy to acoustical energy and vice versa are housed inside sonar domes that are designed to withstand the mechanical loads they encounter and have good acoustical transparency. The thickness of the dome is always much less than the radius of curvature of the dome. Acoustic waves from a far are plane when they reach the dome, but the wave that is transmitted into the dome is not plane. This occurs because the dome is doubly curved and the angle of incidence is a function of the coordinates of the dome.

In this paper, a ray approach is presented to determine the acoustic field when a plane acoustic wave is incident on a solid layer embedded in an infinite fluid. Ray theory is used to determine the reflected and transmitted waves and the field inside the solid layer. When a ray is incident on a solid-fluid interface, two types of rays within the solid layer and one in the fluid are generated. The rays in the solid known as dilatational and distortional rays travel at different speeds. Each ray that is generated undergoes multiple reflections and transmissions. A method is presented to systematically account for all the rays and find the sum of their contributions. The magnitude and phase of the fields, for oblique incidence of the wave, obtained using the ray approach, are compared with those obtained using wave theory.

II. Statement Of The Problem

Consider a thin solid layer of infinite lateral extent embedded in an infinite fluid as shown in Fig. 1. The normal to the layer is along the x axis as shown in the Fig. 1. The layer extends from x = a to x = b, and its thickness, h, is equal to b-a. The media are labeled as I, II, and III for convenience, as shown in Fig. 1.

This work was done at Naval Physical and Oceanographic Lab., Kochi, 682021.

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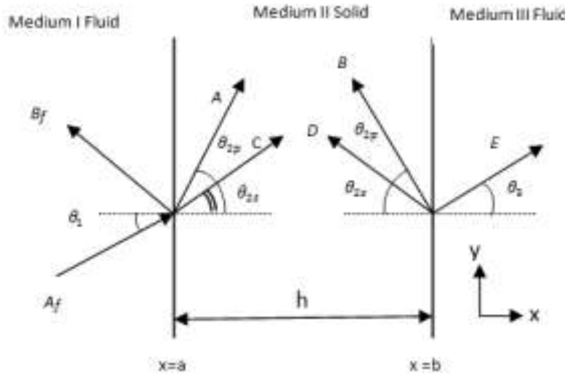

Fig.1 Plane wave incident on thin solid layer embedded in infinite fluid

A plane acoustic wave with angular frequency ω is incident on the layer at an angle θ_1. The normal to the wave lies in the xy plane. The reflected and transmitted velocity potentials and the velocity within the layer are of interest.

The densities of the fluid and solid media are $\rho$and $\rho_s$respectively. The speed of sound in the fluid is$c$. λ and μ are Lame's constants in the solid. Two types of waves are generated within the solid layer: *p*and *s*wavesthat travel at angle $\theta_{2p}$and θ_{2s}, respectively with respect to the x axis. The speeds of the longitudinal (*p*) and transverse (*s*) waves in the solid media are c_p and c_s respectively. A wave is transmitted to the fluid on other side of the plate at an angle θ_3. Since medium I and III are same, θ_3 is equal to θ_1.

Let the velocity potential field in the fluid beΦ_f and the velocity potentialsof the *p*and s waves in the solid layer beϕ and ψ, respectively. Then, the component of velocity in the fluid, normal to the solid layer (x direction), is

$$U_f = \frac{\partial \Phi_f}{\partial x} . \qquad (1)$$

The normal and tangential particle velocities in the solid layer are

$$U = \frac{\partial \Phi}{\partial x} + \frac{\partial \psi}{\partial y} \qquad (2)$$

and

$$V = \frac{\partial \Phi}{\partial y} - \frac{\partial \psi}{\partial x} \qquad (3)$$

respectively.

A ray approach to determine the displacement field is presented.The wave approach is first presented in brief and numerical results obtained using the two approaches are compared to show that the ray analysis is correct.

III. Wave Analysis

The potential field in medium I is the sum of potentials of incident and reflected waves. Hence the total velocity potential in the first medium is expressed as

$$\Phi_f = \left[A_f e^{(-j\alpha_f x)} + B_f e^{(j\alpha_f x)}\right] e^{(-j\gamma y)} \qquad (4)$$

where $A_f$and B_f are the complex amplitudes of incident and reflected waves respectively. The wave-number of the longitudinal wavein the fluid is $k_f = \omega / c$,$\alpha_f = k_f \cos\theta_1$and $\gamma = k_f \sin\theta_1$are the horizontal and vertical components respectively of the angular wave number k_f .

The velocity potentialsof the *p*and s waves in the solid layer are expressed as

$$\phi = \left[A e^{(-j\alpha x)} + B e^{(j\alpha x)}\right] e^{-j\gamma_p y} \qquad (5)$$

and

$$\psi = \left[C e^{(-j\beta x)} + D e^{(j\beta x)}\right] e^{-j\gamma_s y} \qquad (6)$$

respectively. $A$and C are the complex amplitudes of the potentials of the *p* and *s* waves travelling to the right in the solid;B and D are the complex amplitudes of the potential of the *p*and *s*waves travelling to the left in the solid. The wave-numbers of the *p*and s waves are$k_{2p} = \omega/c_p$and$k_{2s} = \omega/c_s$, respectively. α, β, γ_p and γ_s arecomponents of the angular wave numbers k_{2p} and k_{2s} ,and are defined as $\alpha = k_{2p}\cos\theta_{2p}$, $\beta = k_{2s}\cos\theta_{2s}$,$\gamma_p = k_{2p}\sin\theta_{2p}$ and $\gamma_s = k_{2s}\sin\theta_{2s}$such that $\alpha^2 + \gamma^2 = k_{2p}^2$ and$\beta^2 + \gamma^2 = k_{2s}^2$.

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The potential function associated with waves in the third medium is

$$\Phi_f = Ee^{(-j\,\alpha_f x)}\,e^{(-j\gamma y\,)} \qquad (7)$$

where *E* is the complex amplitude.

The amplitudes and phases of the reflected and transmitted waves in first and third medium and waves within the solid layer are of interest. They are used to determine the displacement.

If the plane wave is transmitted from fluid to solid or solid to fluid, then the normal particle velocity on both fluid and solid sides of the boundary must be equal[2].Therefore

$$\frac{\partial\Phi_f}{\partial x} = \frac{\partial\Phi}{\partial x} + \frac{\partial\psi}{\partial y}\,\text{at}\; x = a \qquad (8)$$

and

$$\frac{\partial\Phi_f}{\partial x} = \frac{\partial\Phi}{\partial x} + \frac{\partial\psi}{\partial y}\,\text{at}\; x = b \qquad (9)$$

The continuity of stress requires that the normal stress in the solid and the pressure in the fluid must be equal at the interface. The pressure in the fluid is

$$P_f = -j\rho\omega\Phi_f \qquad (10)$$

and normal stress in the solid is

$$T = \frac{1}{j\omega}\left((\lambda + 2\mu)\frac{\partial U}{\partial x} + \lambda\frac{\partial V}{\partial y}\right) \qquad (11)$$

Thusat x=a and x=b

$$\frac{1}{j\omega}\left((\lambda + 2\mu)\frac{\partial U}{\partial x} + \lambda\frac{\partial V}{\partial y}\right) = j\rho\omega\Phi_f \qquad (12)$$

In fluid, shear stress is absent. Therefore, the third boundarycondition at the fluid-solid interface is that the shear stress in the solid is equal to zero [1]:

$$\sigma = \frac{\mu}{j\omega}\left(\frac{\partial V}{\partial x} + \frac{\partial U}{\partial y}\right) = 0\text{at x=a and x=b} \qquad (13)$$

These three boundary conditions applied at x=a and x=b interfaces yields six equations, as expressed in matrix form in Eq. (14). The matrix is solved to obtain the amplitude of six velocity potentials, B_f,*A*,*B*,*C*,*D* and *E*.

$$\begin{bmatrix} \alpha_f e^{(j\alpha_f a)} & \alpha e^{(-j\alpha a)} & \gamma e^{(-j\beta a)} & \alpha e^{(j\alpha a)} & \gamma e^{(j\beta a)} & 0 \\ \rho\omega^2 e^{(j\alpha_f a)} & \kappa e^{(-j\alpha a)} & (2\mu\gamma\beta)e^{(-j\beta a)} & \kappa e^{(j\alpha a)} & -2\mu\beta\gamma e^{(j\beta a)} & 0 \\ 0 & (-2\alpha\gamma)e^{(-j\alpha a)} & (\beta^2-\gamma^2)e^{(-j\beta a)} & (2\alpha\gamma)e^{(j\alpha a)} & (\beta^2-\gamma^2)e^{(j\beta a)} & 0 \\ 0 & -\alpha e^{(-j\alpha b)} & \gamma e^{(-j\beta b)} & -\alpha e^{(j\alpha b)} & -\gamma e^{(j\beta b)} & \alpha_f e^{(-j\alpha_f b)} \\ 0 & \kappa e^{(-j\alpha b)} & (2\mu\gamma\beta)e^{(-j\beta b)} & \kappa e^{(j\alpha b)} & (\beta^2-\gamma^2)e^{(j\beta b)} & -\rho\omega^2 e^{(-j\alpha_f b)} \\ 0 & (-2\alpha\gamma)e^{(-j\alpha b)} & (\beta^2-\gamma^2)e^{(-j\beta b)} & (2\alpha\gamma)e^{(j\alpha b)} & (\beta^2-\gamma^2)e^{(j\beta b)} & 0 \end{bmatrix}\begin{bmatrix} B_f \\ A \\ C \\ B \\ D \\ E \end{bmatrix} = \begin{bmatrix} \alpha_f e^{(-j\alpha_f a)} \\ \rho\omega^2 e^{(-j\alpha_f a)} \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \qquad (14)$$

where$\kappa = \left(\lambda k_{2p}^2 + 2\mu\alpha^2\right)$

Power balance and special cases are considered below to show that Eq. (14) is correct.

A. Power balance

The matrix Eq. (14) and the numerical results obtained by solving it should satisfy certain power balance checks. First, consider two vertical planes parallel to the surfaces of the solid layer. One plane is in fluid medium I and the other is within the solid layer. The input acoustic power to the region between these two planes is due to waves with amplitudes A_f, B and D. The output power is due to waves with amplitudes B_f,A and C. As the problem is defined as a no-loss case, input power must be equal to output power. Second, consider two vertical planes parallel to the surfaces of the solid layer. One plane is in fluid medium I and the other is in fluid medium III. Input power is due to the incident wave. Output power is due to reflected waves at x=a and transmitted waves at x=b. Input power must be equal to output power. Hence $T_\pi + R_\pi = 1$where, T_π is the power transmission coefficient and R_π is the power reflection coefficient [1]. These two power balance checks are satisfied by the solutions to the equation.

B. Special cases

The results for certain special cases are obtained by using the matrix Eq. (14); and this proves that the Eq. (14) and the solutions are correct. The first case is a thin fluid layer embedded in same fluid subjected to normal incidence by a plane acoustic wave. Hence there is complete transmission and no reflection [1].By assigning values $\theta = 0$, $c_s = 0$,$c_p = c$, $\rho_s = \rho$, $\mu=0$ to the parameters in Eq. (14), it reduces to

$$\begin{bmatrix} \alpha_f e^{(j\alpha_f a)} & \alpha e^{(-j\alpha a)} & \alpha e^{(j\alpha a)} & 0 \\ \rho\omega^2 e^{(j\alpha_f a)} & (\lambda k_{2p}^2)e^{(-j\alpha a)} & (\lambda k_{2p}^2)e^{(j\alpha a)} & 0 \\ 0 & -\alpha e^{(-j\alpha b)} & -\alpha e^{(j\alpha b)} & \alpha_f e^{(-j\alpha_f b)} \\ 0 & (\lambda k_{2p}^2)e^{(-j\alpha b)} & (\lambda k_{2p}^2)e^{(j\alpha b)} & -\rho\omega^2 e^{(-j\alpha_f b)} \end{bmatrix} \begin{bmatrix} B_f \\ A \\ B \\ E \end{bmatrix} = \begin{bmatrix} \alpha_f e^{(-j\alpha_f a)} \\ \rho\omega^2 e^{(-j\alpha_f a)} \\ 0 \\ 0 \end{bmatrix} \qquad (15)$$

The solutions to Eq. (15) are B_f =0, A=1, B=0, and E=1.B_f =0, B=0 implies that there is no reflection. A=1 and$E$=1 implies that complete transmission occurs at both the interfaces. These are the expected results. As a second case, consider oblique incidence on a thin fluid layer embedded in the same fluid. Again, complete transmission is expected and it is indeed the case. Third, consider normal incidence of a plane wave on a fluid layer embedded in a different fluid. By using $\theta =0$, $c_s= 0$,$c_p \neq c$, $\rho_s \neq \rho$, $\mu=0$,Eq. (14) reduces to the set of 4 equations in Ref. 1.

When a wave is obliquely incident on an interface between two semi-infinite fluids, 1+R=T where R and T are reflection and transmission coefficients, respectively [1]. When a wave is obliquely incident on a thin fluid layer embedded in different fluid, |1+R|= |T| where | | denotes magnitude. Using $\theta \neq 0$, $c_s = 0$ and $c_p \neq 0$, $\rho_s \neq \rho$, $\mu=0$in Eq. (14) yields Eqs. whose solutions satisfy the condition.

When a plane wave is normally incident on a thin solid layer embedded in fluid, only longitudinal waves are generated in solid layer [5]. For this case, the solutions C and D to Eq. (14) are zero. Therefore, the condition is satisfied.

When a plane wave is obliquely incident on a thin solid layer embedded in fluidand the angle of incidence, $\theta_1 = \sin^{-1}(c/(\sqrt{2}\, c_s))$, then θ_{2s} =45^0, and only transverse waves are generated in solid [2]. The solutions to Eq. (14) satisfy this condition.

IV. Ray Analysis

The assumption that energy is carried along paths that rays take through the medium is used to describe the acoustic field. To define the ray, the local angle that it makes with the global x axis, θ, is specified. The phase of the ray is expressed in a coordinate system that is attached to the ray. The x axis of the local coordinate system is along the direction of the wave.

When a *f* ray (longitudinal ray in fluid) is incident on the fluid- solid interface a *f* ray get reflected and a *p* and a *s* ray get transmitted. When a *p* or *s* ray reaches the solid- fluid interface both *p* and *s* rays get reflected and a *f* ray gets transmitted and travels to infinity.This happens whenever a ray meets an interface and can take place infinite number of times before the ray reaches the field point. The total acoustic field is the sum of all rays.

A ray is labelled as n=1 ray when the path travelled by the ray within the solid layer has only one segment. That is, the ray has undergone no reflection when travelling within the layer. Similarly a ray is labeled n=2 ray when the path travelled by the ray within the solid layer involves two segments. That is, the ray has undergone one reflection when travelling within the layer. The segments may be *p* or *s* segments. It is necessary to distinguish between the four n=2 rays. Therefore, they are labeled as *2pp, 2ps, 2sp,* and *2ss* where the nth letter indicates the type of the nth segment. In general, a ray with n segments has 2^n labels. For example, all the following are n=3 rays: *3ppp, 3pps, 3pss, 3spp, 3ssp, 3sps, 3psp, 3sss*.The total acoustic field is due to the n= 0, 1, 2, … rays.

The rays that emerge from the layer and travel in the fluid in which the incident wave is travelling are labeled such that it is possible to identify each segment of that ray. For example, consider the ray 2psf. The first segment is a p ray that is transmitted at the fluid-solid interface. The second segment is a s ray that is reflected at the solid-fluid interface. The third segment is a f ray that is transmitted at the solid-fluid interface. The total field that is reflected from the layer has contributions only from n = 0, 2, 4, … rays. The total field that is transmitted through the layer has contributions only from n = 1, 3, 5, … rays. The internal field in the layer has contributions from n = 1, 2, 3, … rays.

The potential of any ray can be expressed as

$$\phi_n = A_n e^{j\zeta_n} \quad (16)$$

where A_n is the amplitude of the ray which has undergone n-1 reflections and ζ_n is the phase. $e^{j\omega t}$ is the variation in time, of the potential. Since this term appears in all expressions, it is suppressed in this equation and will be, for all other equations. Medwin and Clay computed the reflection and transmission coefficients of a thin fluid layer embedded in an infinite fluid by considering the total up travelling signals as the sum of an infinite number of partial transmissions and reflections. Total reflection and transmission coefficients were obtained by solving a geometric series. This geometric series and analytical solution can be applied only to a fluid layer. In a solid layer due to the coupling between the p and s waves the number of rays to track, and in turn the number of reflection and transmission calculations, increases exponentially with each internal reflection.

Here a numerical solution is obtained by adding the potentials due to several rays and convergence is verified by comparing the results with those obtained using wave theory. For the sonar dome, wave analysis is not possible and convergence can be verified by increasing the number of terms used to find the sum.

C. Semi-infinite Media: Reflection and Transmission Coefficients

In the ray approach, the reflection and transmission coefficients when *f, p*, and *s* rays meet an interface at x =0 between two semi-infinite media are used.

When a longitudinal wave is incident on a fluid-sold interface, the continuity conditions lead to

$$\begin{bmatrix} \alpha_f & -\gamma & \alpha \\ -\rho\omega^2 & \kappa & (2\mu\gamma\beta) \\ 0 & (-2\alpha\gamma) & (\beta^2-\gamma^2) \end{bmatrix} \begin{bmatrix} R_{ff} \\ T_{fp} \\ T_{fs} \end{bmatrix} = \begin{bmatrix} \alpha_f \\ \rho\omega^2 \\ 0 \end{bmatrix} (17)$$

R_{ff} is the reflection coefficient for the longitudinal wave, T_{fs} is the transmission coefficient for the s wave, and T_{fp} is the transmission coefficient for the p wave.

When a p ray is incident at the solid-fluid interface the continuity conditions lead to

$$\begin{bmatrix} \alpha & -\gamma & \alpha_f \\ \kappa & -(2\mu\gamma\beta) & -\rho\omega^2 \\ 2\alpha\gamma & (\beta^2-\gamma^2) & 0 \end{bmatrix} \begin{bmatrix} R_{pp} \\ R_{ps} \\ T_{pf} \end{bmatrix} = \begin{bmatrix} \alpha \\ \kappa \\ 2\alpha\gamma \end{bmatrix} (18)$$

where R_{ps} is the reflection coefficient for the s ray,R_{pp} is the reflection coefficient for the p ray, and T_{pf} is the transmission coefficient for the wave in the fluid.

When a s ray is incident at the solid-fluid interface the continuity conditions lead to

$$\begin{bmatrix} \alpha_f & -\gamma & \alpha \\ \kappa & -(2\mu\gamma\beta) & -\rho\omega^2 \\ 2\alpha\gamma & (\beta^2-\gamma^2) & 0 \end{bmatrix} \begin{bmatrix} R_{sp} \\ R_{ss} \\ T_{sf} \end{bmatrix} = \begin{bmatrix} \gamma \\ -2\mu\gamma\beta \\ -(\beta^2-\gamma^2) \end{bmatrix} (19)$$

T_{sf} is the transmission coefficient, R_{ss} is the reflection coefficient for the s ray, and R_{sp} is the reflection coefficient for the p ray.

Eqs. (17) to (19) are solved for the particular angle of incidence. The results are used in the ray analysis.

D. Solid Layer: Reflection coefficient

When a plane acoustic wave of unit amplitude is incident on the solid layer, the potential reflection coefficient is the sum of potentials of all the rays emerging out to first medium at *x=a* interface as shown in Fig.2.

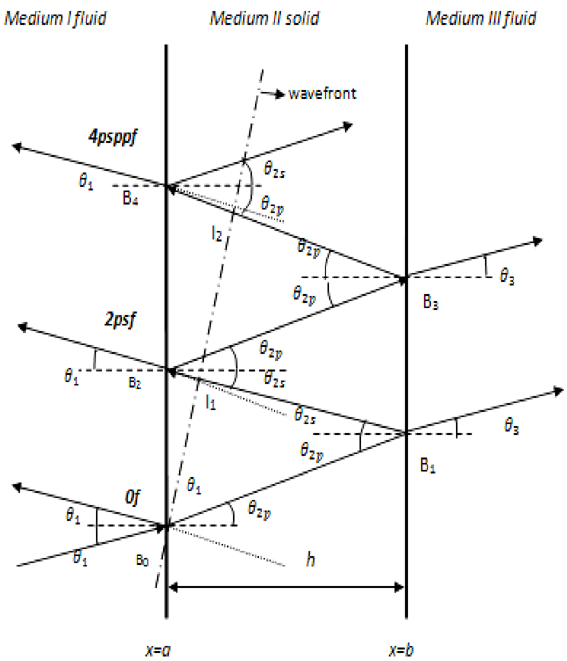

Fig. 2.Schematic of ray propagation through the solid layer with imaginary wave front for reflected rays at x=a interface

It is expressed as

$$R_a = \sum_{n=0,2,4..}^{\infty} \phi_{nf} \qquad (20)$$

where ϕ_{nf} are the potentials of emerging rays. *n* indicates the number of ray segments within the solid layer. Each ϕ_{nf} term indicates the sum of all $2^n p$ and *s* combinations of rays. For example, $\phi_{2f} = \phi_{2ppf} + \phi_{2psf} + \phi_{2spf} + \phi_{2ssf}$.The general expression for ϕ_{nf} is

$$\phi_{nf} = A_{nf}\, e^{j(k_1 \xi_n - \eta_n)} \qquad (21)$$

In Fig. 2, the reflected rays are 0f, 2psf, 4psppf etc.

The amplitude of any ray emerging from the solid layer into the fluid in the left is expressed as

$$A_{nf} = \left[(T_{fs} a + T_{fp} b)(R_{ss})^c (R_{sp})^d (R_{ps})^e (R_{pp})^f (T_{sf} g + T_{sf} l)\right] \qquad (22)$$

where n is an even number. *a, b, c, d, e, f, g,l* are the number of the corresponding events occurring in the considered ray. It is to be noted that *a, b, g and l* acquires values only zero and one. Also when *a* is one, *b* is zero and when *g* is one, *l* is zero and vice versa. It is because for a given ray only the first segment is transmitted at fluid- solid interface and last segment at solid-fluid interface. This is applicable for all succeeding cases explained in this chapter.

ξ_n corresponds to the path not travelled by a particular ray with respect to origin at an instant. A wave-front is defined as the surface on which phase is constant. It follows that the wave-front, in 2D space, at any instant of time, is a straight line and will be perpendicular to direction of propagation. An imaginary wave-front is drawn for the reflected rays such that it passes through the origin. Thus for 2psf ray ξ is distance B_2I_1 in Fig 2. denoted as $\overline{B_2 I_1}$,for ray 4psppf,$\overline{B_4 I_2}$ and so on.

The phase term is generalized as

$\mathrm{k}_1\xi_n - \eta_n = \mathrm{h}\left[N_p k_{2p} \cos\theta_{2p} + N_s k_{2s} \cos\theta_{2s}\right]$(23)

where N_p and N_s are number of p and s segments in a given ray respectively. Thus the total reflection coefficient at first interface,

$$\mathrm{R_a} = R_{ff} + \sum_{n=2,4..}^{\infty} A_n\, e^{j(\mathrm{k}_1\xi_n - \eta_n)} \qquad (24)$$

$\mathrm{R_a}$ corresponds B_f .

E. Solid Layer: Rays within the Layer

The effect of all the rays travelling within the layer is a standing *p* wave and a standing *s* wave. Each standing wave is comprised of one wave travelling to the right and another travelling to the left. It is of interest to determine the complex amplitude of the *p* and *s* waves travelling to the right and left.

a) *Waves travelling to the right*

The p wave travelling to the right is comprised of all n = 1, 3, 5 … rays ending with a p segment. Amplitude of its potential, *A* is expressed as

$$A = \sum_{n=1,3,5..}^{\infty} \phi_{n*p} \qquad (25)$$

where *p indicates that any sequence of rays that ends with a p segment is to be considered. The s wave traveling to the right is comprised of all n = 1, 3, 5 … rays ending with a s segment. Its potential, C is expressed as

$$C = \sum_{n=1,3,5.}^{\infty} \phi_{n*s} \qquad (26)$$

From Fig.3 it is evident that rays, 1p, 3psp, 5pspps….travel to rightfrom first interface.
The general expression for ϕ_{n*p} and ϕ_{n*s} are

$$\phi_{n*p} = A_n e^{j(k_{2p}\xi_n - \eta_n)} \qquad (27)$$

$$\phi_{n*s} = A_n e^{j(k_{2s}\xi_n - \eta_n)} \qquad (28)$$

The amplitude of any ray(p or s) travelling to right at first interface is given by

$$A_n = \left[(T_{fs}a + T_{fp}b)(R_{ss})^c (R_{sp})^d (R_{ps})^e (R_{pp})^f\right] \quad (29)$$

where n is an odd number. *a, b, c, d, e, f, g* are the number of the corresponding events occurring in the considered ray.

Imaginary wave fronts of p and s rays are drawn such that they passes through the origin .ξ depends on last segment of the ray since the wave fronts of *p* and *s* waves are different.

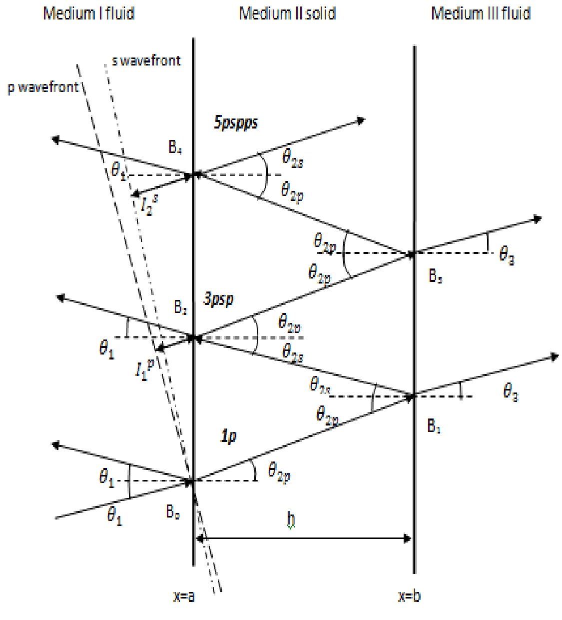

Fig. 3. Schematic of ray propagation through the solid layer with imaginary wave front for p and s rays travelling right at x=a interface.

In general, phase for p rays travelling right is given by
$k_{2s}\xi_n - \eta_n = -\mathrm{h}\left[\left(N_p - 1\right)k_{2p}\cos\theta_{2p} + N_s k_{2s}\cos\theta_{2s}\right]$ (30)
Phase for s rays travelling right is given by
$k_{2s}\xi_n - \eta_n = -\mathrm{h}\left[N_p k_{2p}\cos\theta_{2p} + (N_s - 1)k_{2s}\cos\theta_{2s}\right]$ (31)

b) Waves travelling to the left

The complex amplitude of *p* and *s* waves travelling to leftfrom second interface is also computed in similar way. All n = 2,4,6…rays travel to the left within the solid layer as shown in Fig.4
Potential of pand *s* waves travelling to left are expressed as

$$B = \sum_{n=2,4,6\ldots}^{\infty}\phi_{n*p} \quad (32)$$

$$D = \sum_{n=2,4,6\ldots}^{\infty}\phi_{n*s} \quad (33)$$

The general expression for ϕ_{n*p} and ϕ_{n*s} are

$$\phi_{n*p} = A_n e^{j\,(k_{2p}\xi_n - \eta_n - \chi)} \quad (34)$$

$$\phi_{n*s} = A_n e^{j\,(k_{2s}\xi_n - \eta_n - \chi)} \quad (35)$$

The amplitude of any ray travelling left at second interface is given by
$A_n = \left[(T_{fs}a + T_{fp}b)(R_{ss})^c\left(R_{sp}\right)^d\left(R_{ps}\right)^e\left(R_{pp}\right)^f\right]$ (36)
where n is even.

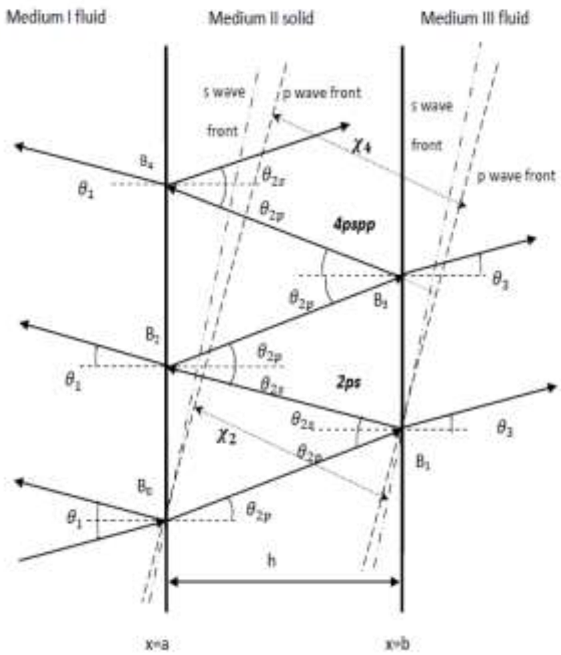

Fig. 4. Schematic of ray propagation through the solid layer with imaginary wave fronts for p and s rays travelling to left are shifted to at x=a interface.

The imaginary wave-fronts are drawn such that they passes through the point of intersection of first reflected p or s ray with x=b interface. Wave-fronts have to be shifted to origin.χ corresponds to the distance through which the wave fronts are shifted to origin multiplied by the wave number of the ray. Value of χ depends on the initial and final segments of the considered ray .
Other components of phase are computed as before. General form of phase is
$(k_2\xi_n - \eta_n - \chi) = -h\left[N_p k_{2p}\cos\theta_{2p} + N_s k_{2s}\cos\theta_{2s}\right]$ (37)

where N_p and N_s are number of p and s segments in a given ray respectively.

*F. **Solid Layer: Transmission Coefficient***

The transmission coefficient at second interface will be the sum of all rays getting transmitted at x=b. From Fig.5 it is evident that all odd set of rays, 1pf, 3pspf,5pspppf….get transmitted at second interface. Transmission coefficient will be the sum of potential of all such rays.

$$T_b = \sum_{n=1,3..}^{\infty} \phi_{nf} \qquad (38)$$

The general expression for ϕ_{nf} is

$$\phi_{nf} = A_{nf}\, e^{j(k_2\xi_n - \eta_n + \chi)} \qquad (39)$$

The amplitude of ray is given by

$$A_n = \left[(T_{fs}a + T_{fp}b)(R_{ss})^c (R_{sp})^d (R_{ps})^e (R_{pp})^f (T_{sf}g + T_{sf}l)\right] \qquad (40)$$

for odd values of n.

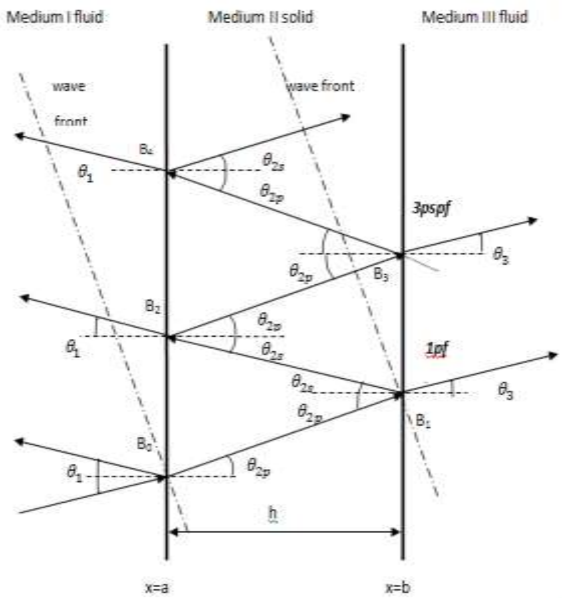

Fig. 5. Schematic of rays transmitted through the solid layer with imaginary wave fronts fortransmitted rays are shifted to at x=a interface.

The wave-front has to be shifted to origin. The value of χ depends on the first segment of the ray. If it is a p segment then

$$\chi = k_f h \frac{\cos(\theta_3 - \theta_{2p})}{\cos\theta_{2p}} \qquad (41)$$

If first segment is s,

$$\chi = k_f h \frac{\cos(\theta_3 - \theta_{2s})}{\cos\theta_{2s}} \qquad (42)$$

Phase term can be generalized as

$$(k_2\xi_n - \eta_n + \chi) = -h\left[N_p \cos\theta_{2p} + N_s k_{2s}\cos\theta_{2s} - k_f \cos\theta_3\right] \qquad (43)$$

Thus the total transmission coefficient at x=b

$$T_b = \sum_{n=1,3,5..}^{\infty} A_{nf}\, e^{j(k_2\xi_n - \eta_n + \chi)} \qquad (44)$$

And corresponds to E_f in wave analysis.

V. Numerical Results And Discussions

G. Normal Incidence

Numerical results are presented for fluid of density ρ_1=ρ_3=1000 kg/m^3 and speed of sound c_1=c_3=1500 m/s. The solid has density ρ_s=1500 kg/m^3. The speed of the p and s rays are c_p = 2250 m/s and c_s = 882 m/s. The frequency considered is 0 to 5 kHz.

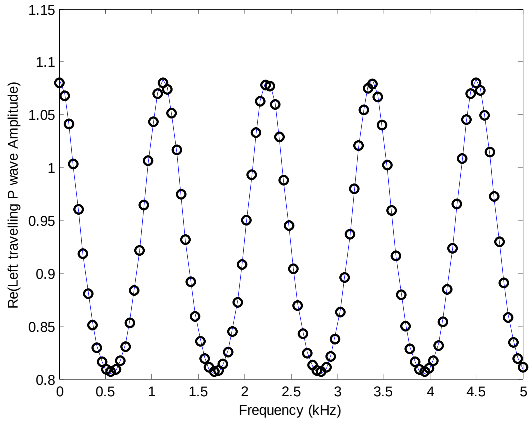

Fig. 6. Amplitude of velocity potential (real part) of left travelling p waves using ray (circles) and wave (line) theory

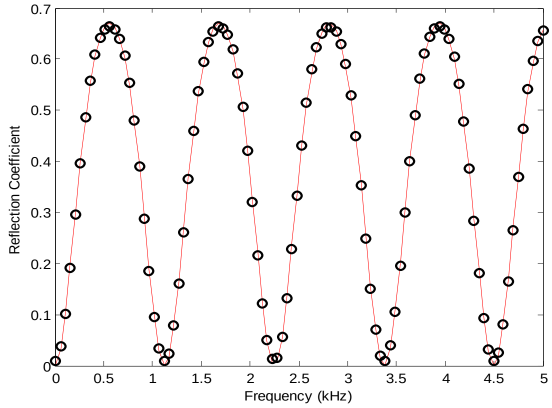

Fig.7 Reflection coefficient (real part) using ray (circles) and wave (line) theory

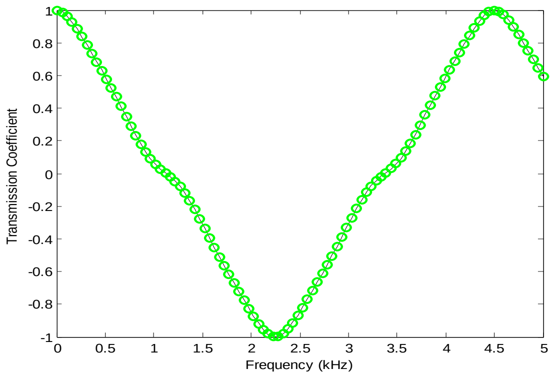

Fig.8. Transmission coefficient (real part) using ray (circles) and wave (line) theory

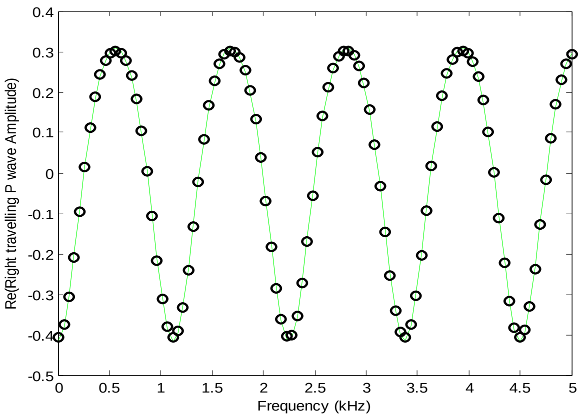

Fig.9 Amplitude of velocity potential (real part) of right travelling p waves using ray (circles) and wave (line) theory

*H. **Oblique Incidence***

Numerical results are presented for fluid of density ρ_1=ρ_3=1000 kg/m^3 and speed of sound c_1=c_3=1500 m/s. The solid has density ρ_s=1500 kg/m^3. The speed of the p and s rays are c_p = 2250 m/s and c_s = 882 m/s. The frequency considered is 1 to 5 kHz.The angle of incidence is 40^o.

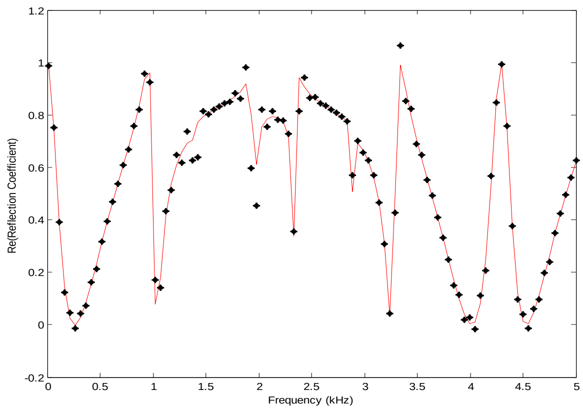

Fig.10, Reflection coefficient (real part) using ray (star) and wave(line)theory

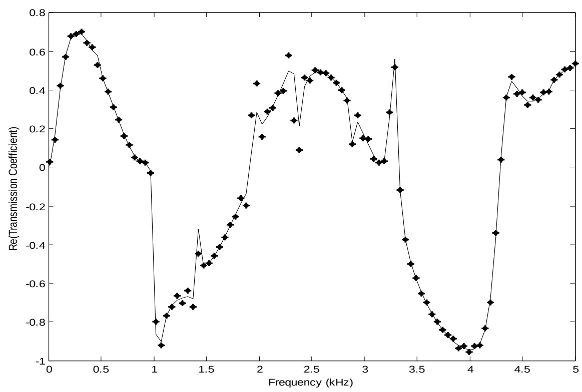

Fig. 11 Transmission coefficient (real part) using ray (star) and wave (line) theory

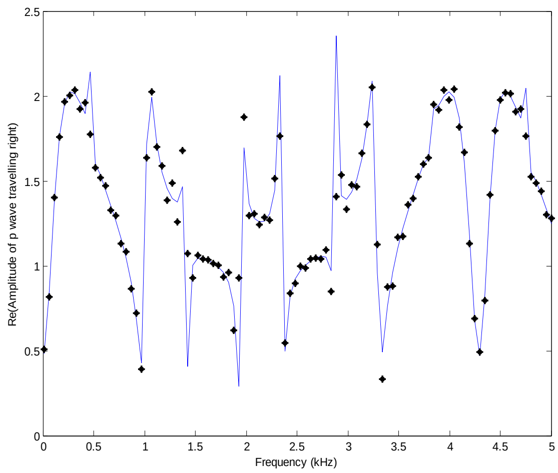

Fig.12 Amplitude of velocity potential (real part) of right travelling p waves using ray (star) and wave (line) theory

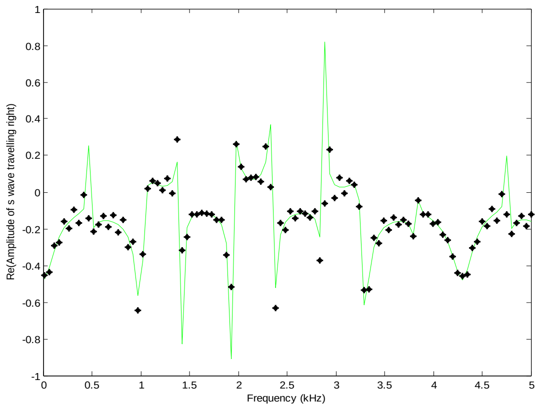

Fig.13 Amplitude of velocity potential (real part) of right travelling s waves using ray (star) and wave (line) theory

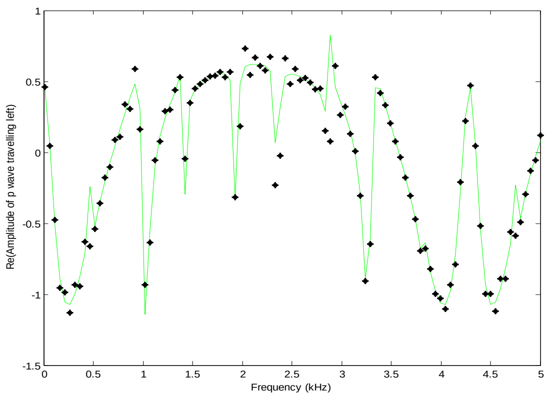

Fig.14 Amplitude of velocity potential (real part) of left travelling p waves using ray (star) and wave (line) theory

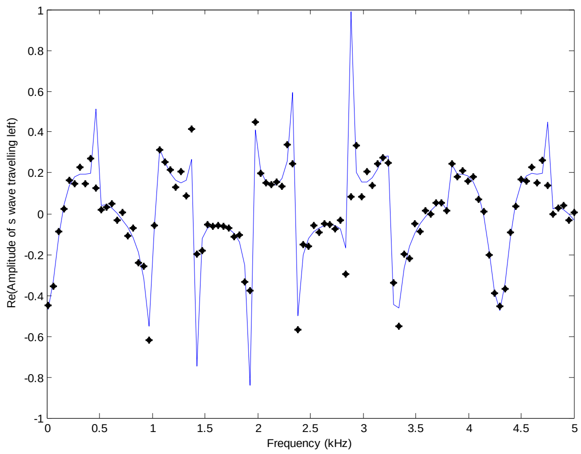

Fig. 15. Amplitude of velocity potential (real part) of left travelling s waves using ray (star) and wave (line) theory

VI. Conclusions

Analysis of transmission through a plane panel is studied using wave theory and ray theory. It is shown that the same results are obtained by using both methods.

The expressions derived for ray fields are general and can be used for analysis of a wave that is obliquely incident on the plane panel that is a thin solid layer embedded in an infinite fluid. A program is written in MATLAB to compute the contributions of 2^n n = 0, 1, 2, … rays to the field. The contributions are summed numerically. For the special case of normal incidence, the results are shown to be in good agreement with analytical results obtained using wave theory. Specifically, the reflection coefficient, the transmission coefficient and complex amplitude of waves within the solid layer computed using ray theory and wave theory are shown to be numerically equal. This shows that the ray approach can be extended to determine the pressure field inside a sonar dome.

REFERENCES

[1] Kinsler, L.E., A.R. Frey, A.B. Coppens, and J.V. Sanders, "Reflection and Transmission," inFundamentals of Acoustics , 4th ed. John Wiley & Sons, 1982, pp.149-169.
[2] L. M. BrekhovskikhAnd O. A. Godin, "Plane wave reflection from the boundary of solids",in Acoustics of Layered Media-I,Springer-Verlag Berlin Heidelberg, New York, 1990, pp. 87-109.
[3] C. S. Clay. and H. Medwin, Acoustical Oceanography: Principles and Applications, John Wiley & Sons, New York, 66-67, 1977.
[4] Folds and Loggins, "Transmission and Reflection of Ultrasonic Waves in Layered Media,",Journal of Acoustical Society of America. 62, pp. 1002-1009 ,1977.
[5] H. Kolsky,"Propagation in an extended elastic medium ,"in Stress Waves in Solids, , Dover Publications, New York 1963,pp.4-40.